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16/ENG05/033

Mechatronics

ENG 281

1a. $\lim_{x \to \pi/2} \left[\frac{(x^2 - \pi/4) \sin(\cos x)}{x - \pi/2} \right]$

Numerator

$u = x^2 - \pi/4$ $\qquad \frac{du}{dx} = 2x$

$v = \sin(\cos x)$ $\qquad$ let $w = \cos x$

$v = \sin w$ $\qquad \frac{dw}{dx} = -\sin x$

$\frac{dv}{dw} = \cos w$

$\frac{dv}{dx} = + \cos \cos x \cdot - \sin x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (x^2 - \pi/4)(\cos \cos x \cdot - \sin x)$
$+ \sin(\cos x)\, 2x$

Denominator

$x - \pi/2$

$\frac{dy}{dx} = 1$

$$\lim_{x \to \pi/2} = \left[\left(\frac{\pi}{2}\right)^2 - \frac{\pi}{4}\right]\left[\cos\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right] + \sin\cos\frac{\pi}{2}\cdot\pi$$

$$= \left[\frac{\pi^2}{4} - \frac{\pi}{4}\right]\left[\cos\cos 90 - \sin 90\right] + \sin\cos 90 \cdot \pi$$

$$= \frac{\pi^2 - \pi}{4} \quad (-1) + (0)$$

$$= \frac{-\pi^2 - \pi}{4}$$

3. $U_n = \dfrac{x^n}{(2n+1)^3}$

$U_{n+1} = \dfrac{x^{n+1}}{(2n+2)^3}$

$$\frac{U_{n+1}}{U_n} = \frac{x \cdot x^n}{(2n+2)^3} \cdot \frac{(2n+1)^3}{x^n}$$

$$= \frac{x(4n^2+4n+1)(2n+1)}{(4n^2+8n+4)(2n+2)}$$

$$= \frac{x(8n^3+12n^2+6n+1)}{8n^3+24n^2+24n+8}$$

$$\lim_{n\to\infty} \frac{8^{n^3}/_{n^3} + 12^{n^2}/_{n^3} + 6n/n^3 + 1/n^3}{8n^3/_{n^3} + 24n^2/_{n^3} + 24n/_{n^3} + 8/n^3}$$

$$= \frac{x(8+0+0+0)}{8+0+0+0}$$

$$= x\ 8/8$$

$$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| = x$$

For absolute convergence $\lim_{n\to\alpha} \left| \frac{u_{n+1}}{u_n} \right| < 1$

$\therefore \leq x \leq 1$ is where x is convergence.

2. Determine whether its convergent.

a. $\frac{2}{2\times3} + \frac{2}{3\times4} + \frac{2}{4\times5} + \frac{2}{5\times6} + \ldots\ldots$

From standard series

$\frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \frac{2}{5^p} + \ldots\ldots$

When p is 2

$\frac{2}{2^2} + \frac{2}{3^2} + 2/4^2 + 2/5^2 + \ldots\ldots$

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Therefore the series is convergent.

b. $\frac{2}{1^2} + \frac{2}{2^2} + \frac{2}{3^2} + \frac{2}{4^2} + \cdots$

For comparision test using standard series when $p = 2$

$$\frac{2}{1^p} + \frac{2}{2^p} + \frac{2}{3^p} + \frac{2}{4^p} + \cdots$$

For comparison test using standard series when $p = 2$.

Since $p > 1$, the series converges

Therefore the series is convergent

c. $U_n = \frac{1 + 2n^2}{1 + n}$

$$\lim_{n \to \infty} U_n = \lim_{n \to \infty} \frac{\left[\frac{1}{n^2} + \frac{2n^2}{n^2}\right]}{\frac{1}{n^2} + \frac{n^2}{n^2}}$$

$$= \left[\frac{\frac{1}{n^2} + \frac{2}{1}}{\frac{1}{n^2} + \frac{1}{1}}\right] = \frac{0+2}{0+1} = \frac{2}{1} = 2$$

Since $\lim_{n \to \infty} a_n \neq 0$ The Series is not convergent.

4. Evaluate using L'Hopital's rule

$$\lim_{x \to 0} \left[\frac{\sin x - \cos x}{x^3} \right]$$

$$= \lim_{x \to 0} \left[\frac{\cos x + \sin x}{3x^2} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\sin x + \cos x}{6x} \right]$$

$$= \lim_{x \to 0} \left[\frac{-\cos x + (-\sin x)}{6} \right]$$

$$= \frac{-\cos 0 - \sin 0}{-6}$$

$$= \frac{-1+0}{6} = -1/6$$